# Angular Visual Hardness

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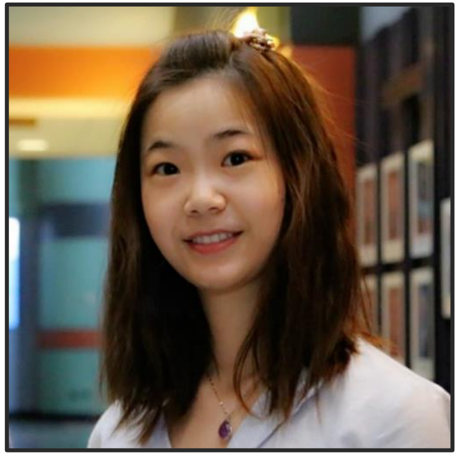
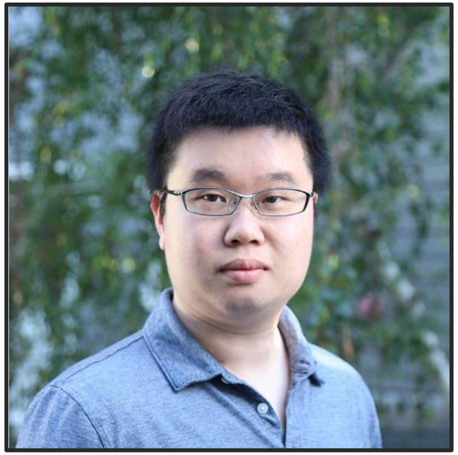


Beidi Chen, Rice

Weiyang Liu, Georgia Tech



Zhiding Yu, NVIDIA

Jan Kautz, NVIDIA

Anshumali Shrivastava, Rice

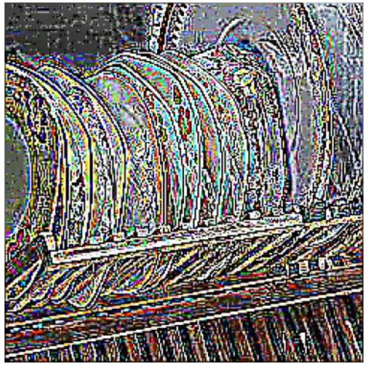
Animesh Garg, NVIDIA

Anima Anandkumar, NVIDIA

# Human Visual Hardness

plate rack

sharpness



contrast

blur

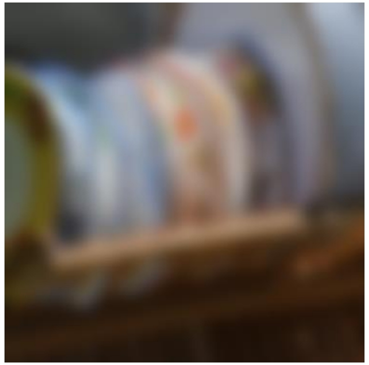


**Image Degradation**

dishwasher

saltshaker

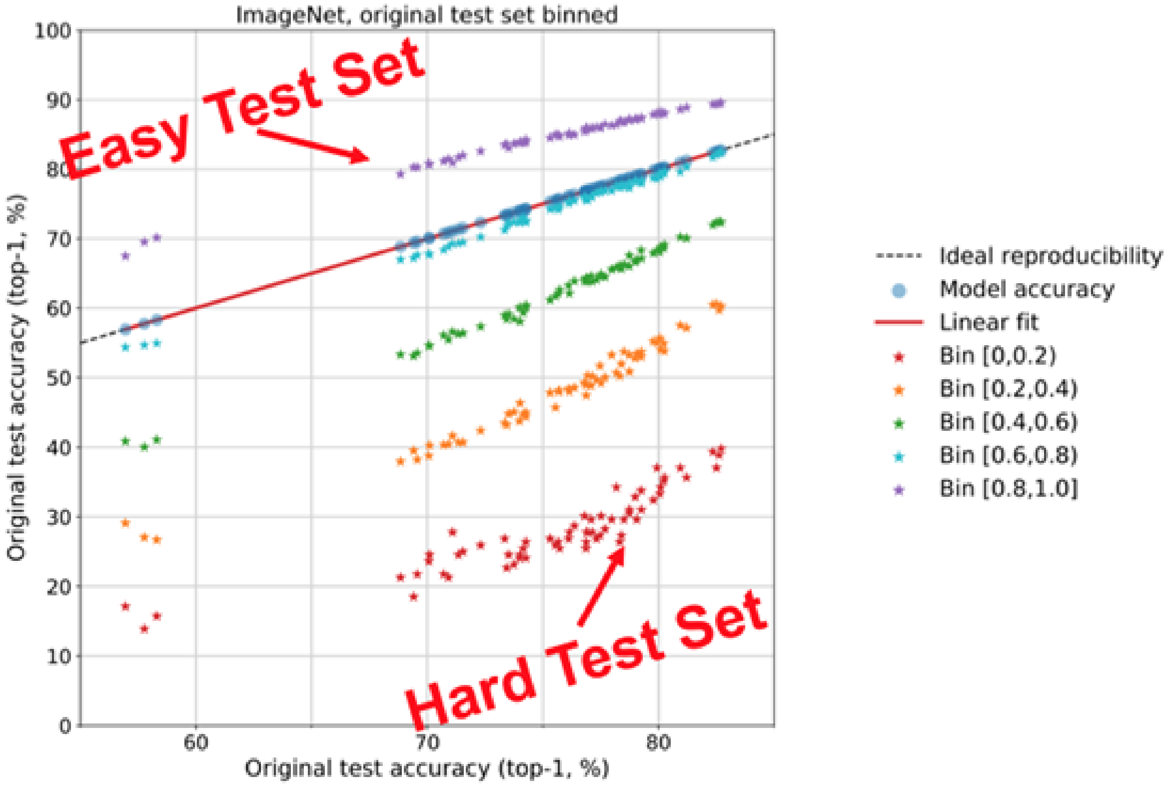
nail

oil filter

**Semantic Ambiguity**

# Human Selection Freq (HSF): A Visual Hardness Proxy

Human Labeling Interface


Recht et al. "Do ImageNet Classifiers Generalize to ImageNet?" ICML 2019

# Gap between Human Recognition and CNNs

**Hard for Human but Easy for CNNs** | **Easy for Human but Hard for CNNs**




| | Nail | Oil Filter | Golf Ball | Radio |
|---|---|---|---|---|
| Softmax | 0.93 | 0.998 | 0.001 | 0.001 |
| HSF | 0.2 | 0.2 | 1.0 | 1.0 |

# Softmax Cross-Entropy Loss

$$L = \frac{1}{N}\sum_i L_i = \frac{1}{N}\sum_i -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right)$$

Magnitude Information

Angle Information

$$L_i = -\log\left(\frac{e^{\|\boldsymbol{W}_{y_i}\|\|\boldsymbol{x}_i\|\cos(\theta_{y_i})}}{\sum_j e^{\|\boldsymbol{W}_j\|\|\boldsymbol{x}_i\|\cos(\theta_j)}}\right)$$

Model Confidence

# Angular Visual Hardness (AVH)

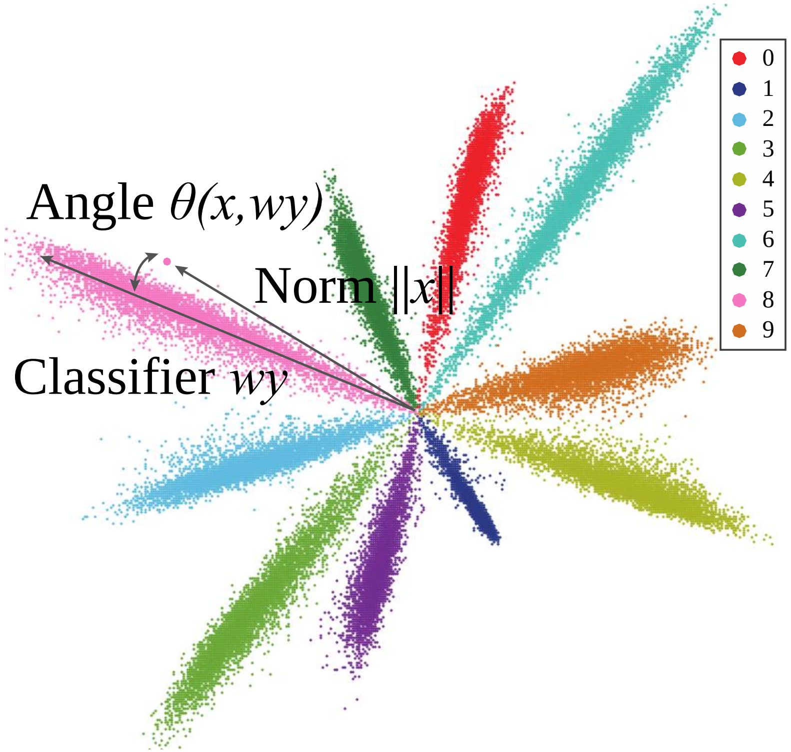
Given a sample $x$ with label $y$:

$$\boldsymbol{AVH}(x) = \frac{\mathcal{A}(x, w_y)}{\sum_{i=1}^{C} \mathcal{A}(x, w_i)}$$

where,

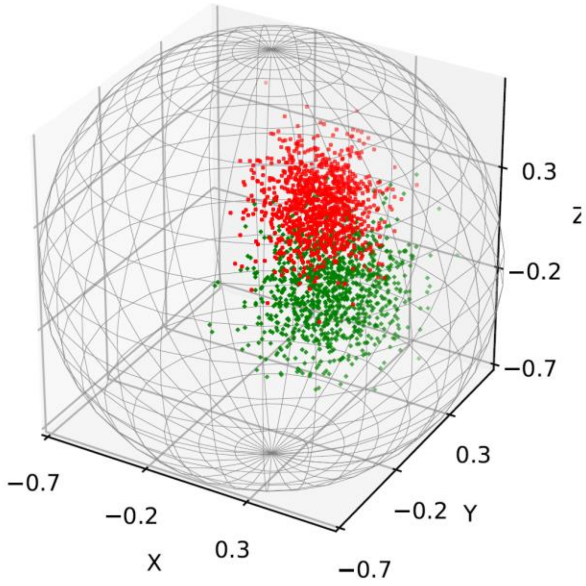
$$\mathcal{A}(\boldsymbol{u}, \boldsymbol{v}) = \arccos\left(\frac{\langle \boldsymbol{u}, \boldsymbol{v} \rangle}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}\right)$$

$w_i$ is the classifier for the $i$-th class.

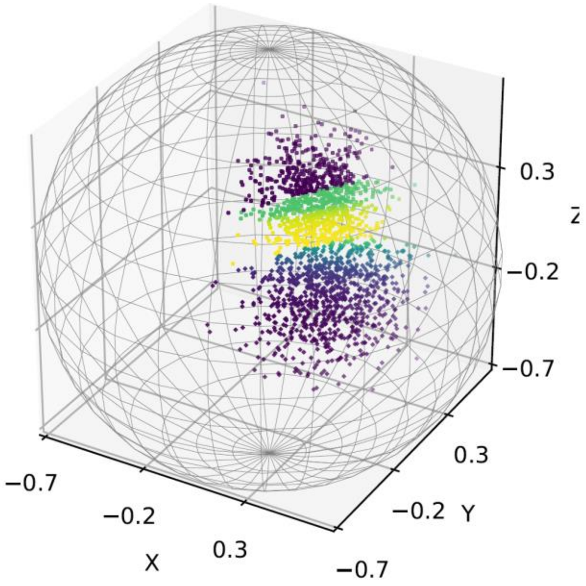



**Theoretical Foundation**:
Soudry et al, The Implicit Bias of Gradient Descent on Separable Data, ICLR18

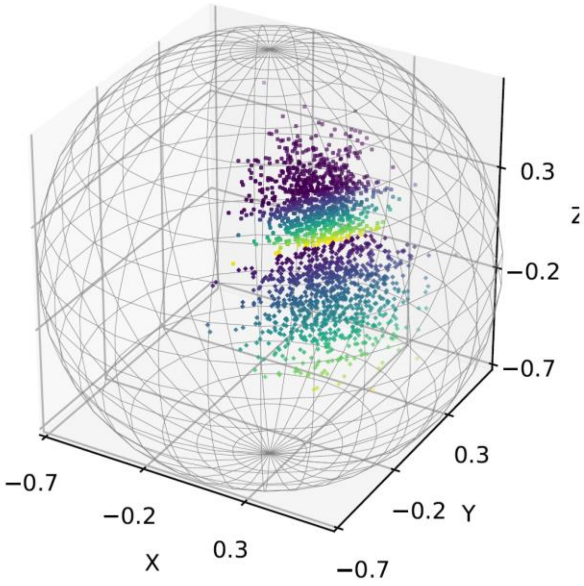
# Toy Example: AVH vs. ||x||




**Raw data**

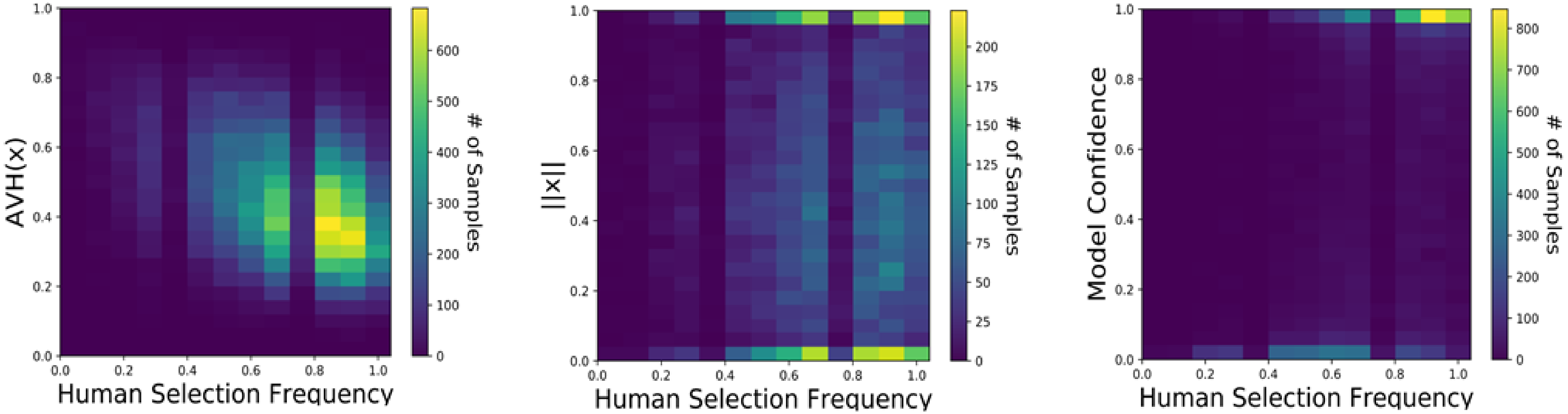



**Heat map of AVH**




**Heat map of ||x||**

# Correlation between Different Measures and HSF

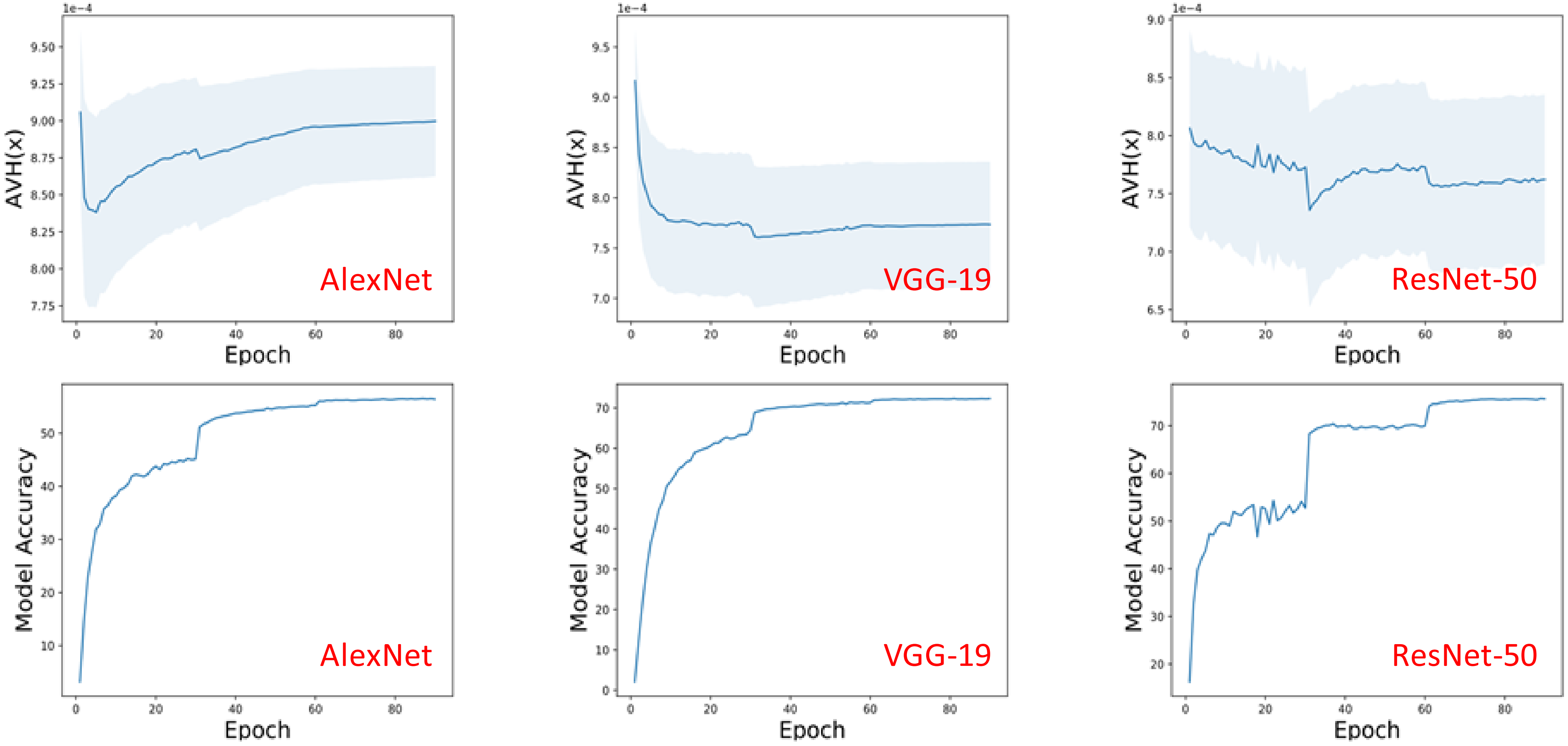



Spearman rank correlations

| | z-score | Total Coef | [0, 0.2] | [0.2, 0.4] | [0.4, 0.6] | [0.6, 0.8] | [0.8, 1.0] |
|---|---|---|---|---|---|---|---|
| Number of Samples | - | 29987 | 837 | 2732 | 6541 | 11066 | 8811 |
| AVH | 0.377 | 0.36 | 0.228 | 0.125 | 0.124 | 0.103 | 0.094 |
| Model Confidence | 0.337 | 0.325 | 0.192 | 0.122 | 0.102 | 0.078 | 0.056 |
| $\|\|\mathbf{x}\|\|_2$ | - | 0.0017 | 0.0013 | 0.0007 | 0.0005 | 0.0004 | 0.0003 |

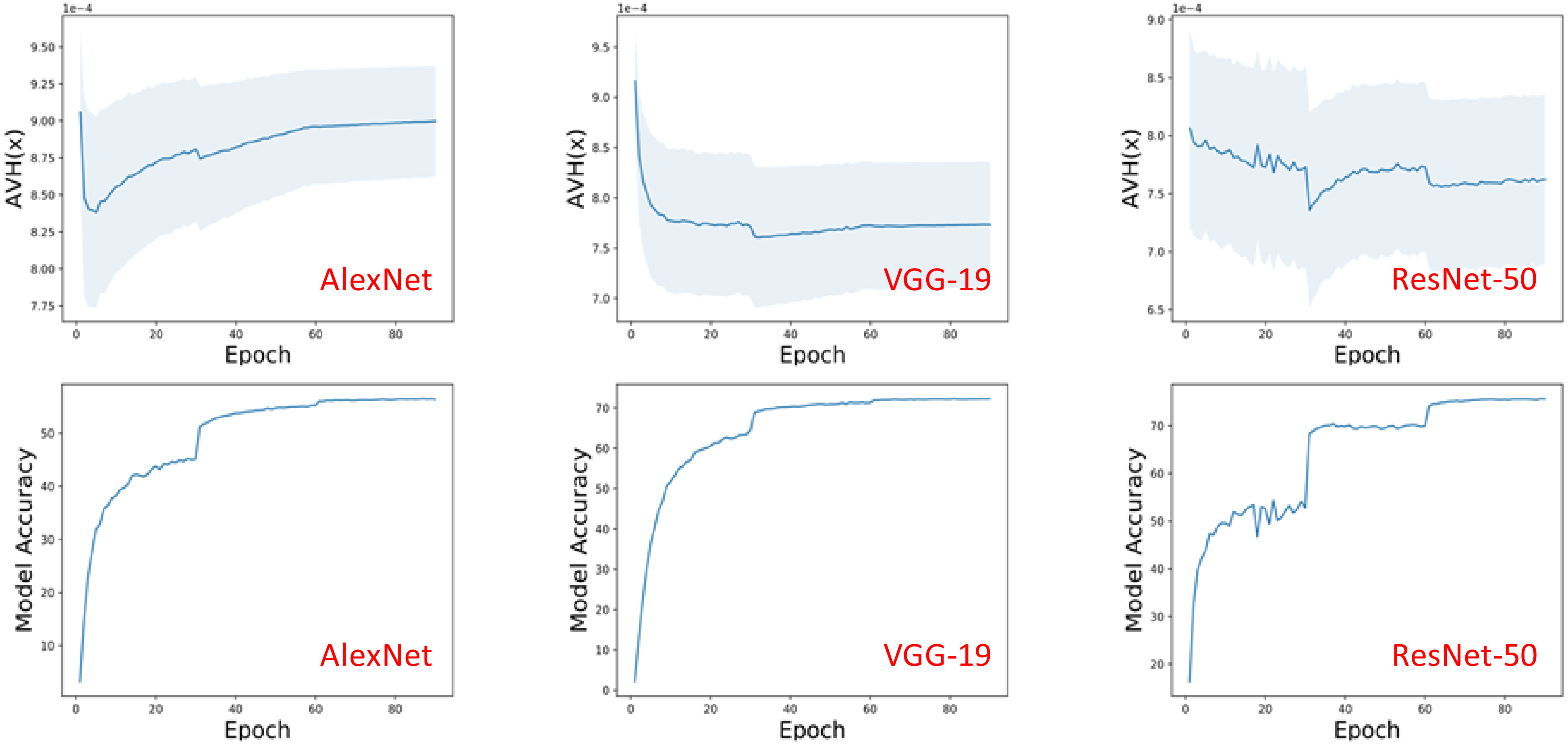
# Main Discoveries

**Discovery 1 -** AVH hits plateau early even though accuracy or loss is still improving



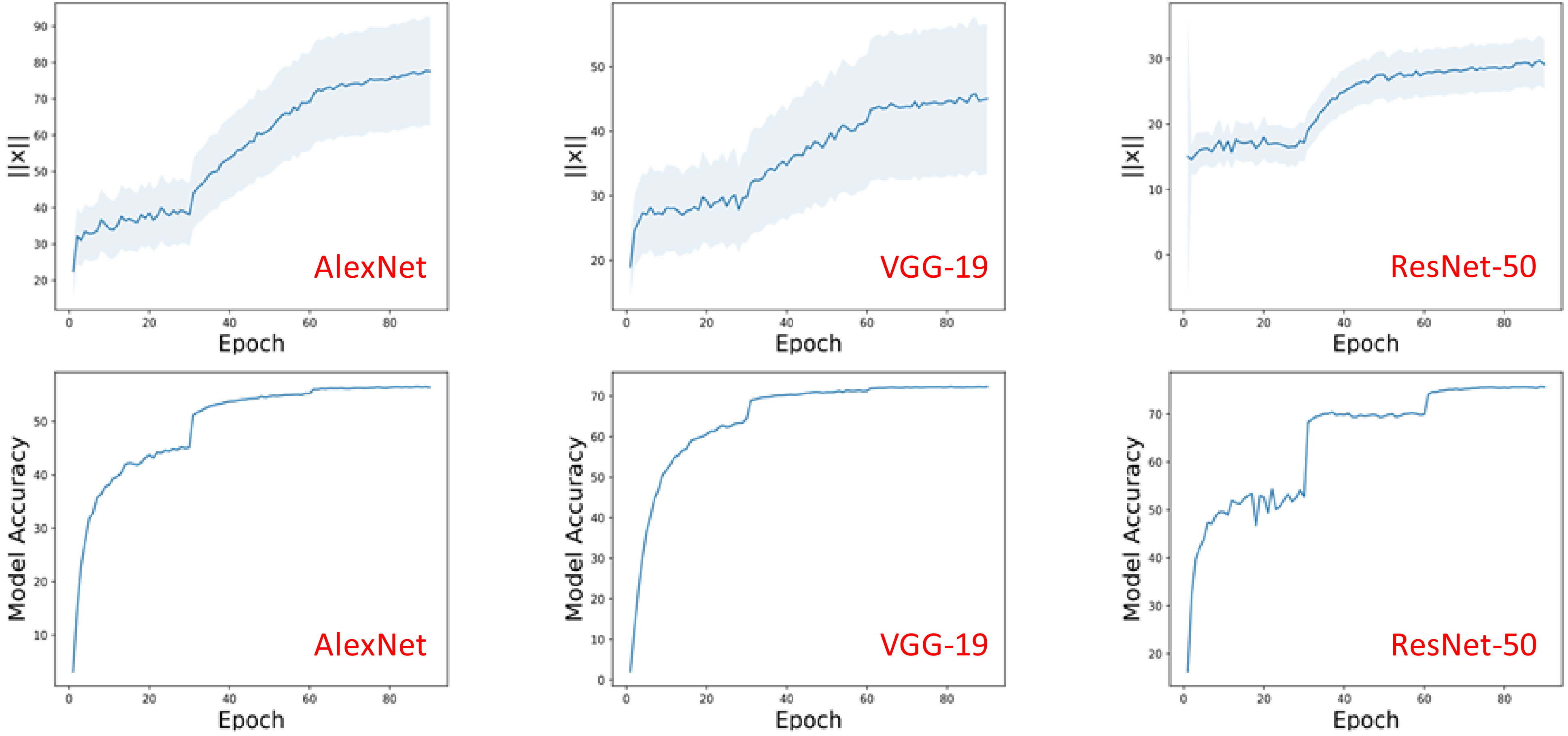
# Main Discoveries

**Discovery 2 -** AVH is an indicator of model's generalization ability



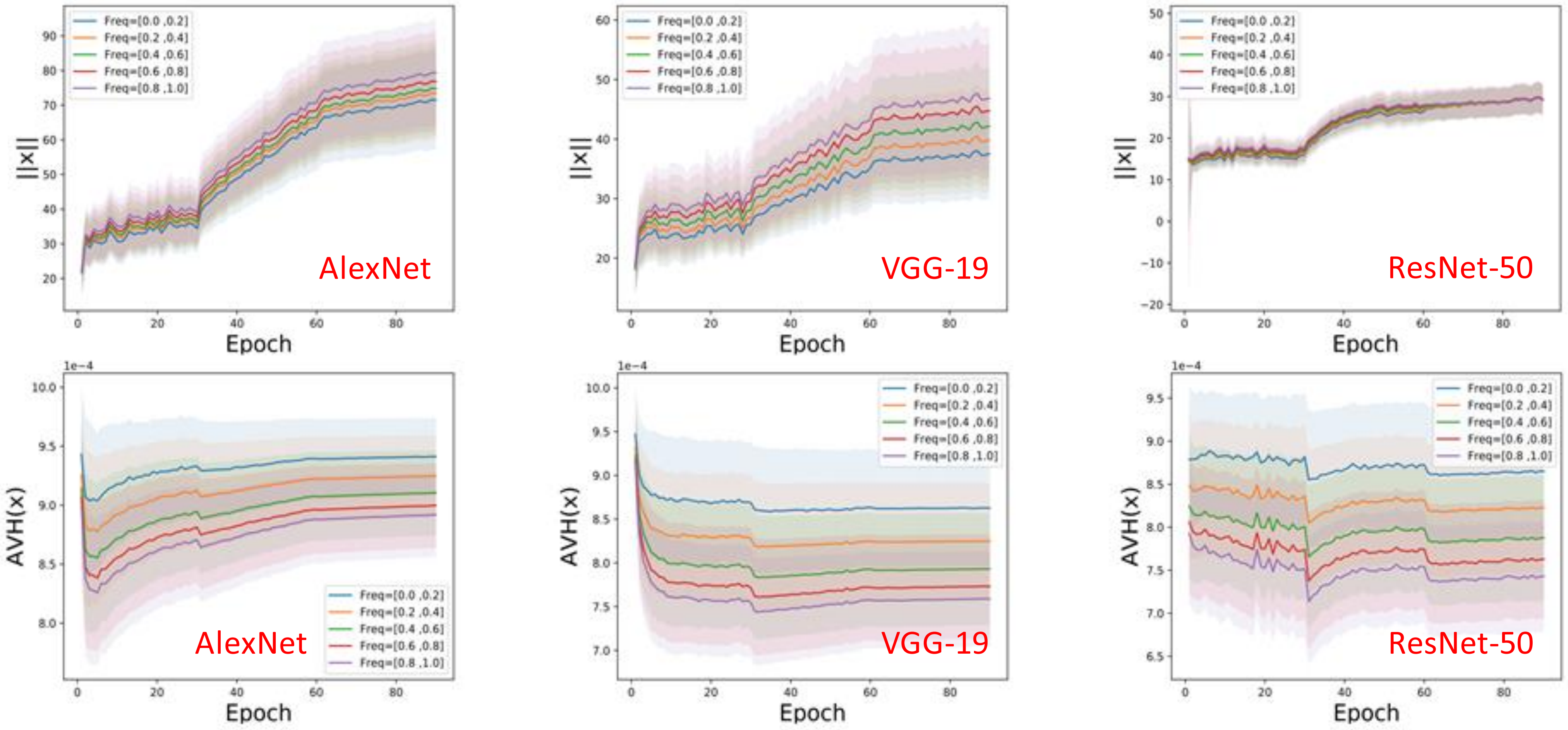
# Main Discoveries

**Discovery 3** - The norm of feature embeddings keeps increasing during training



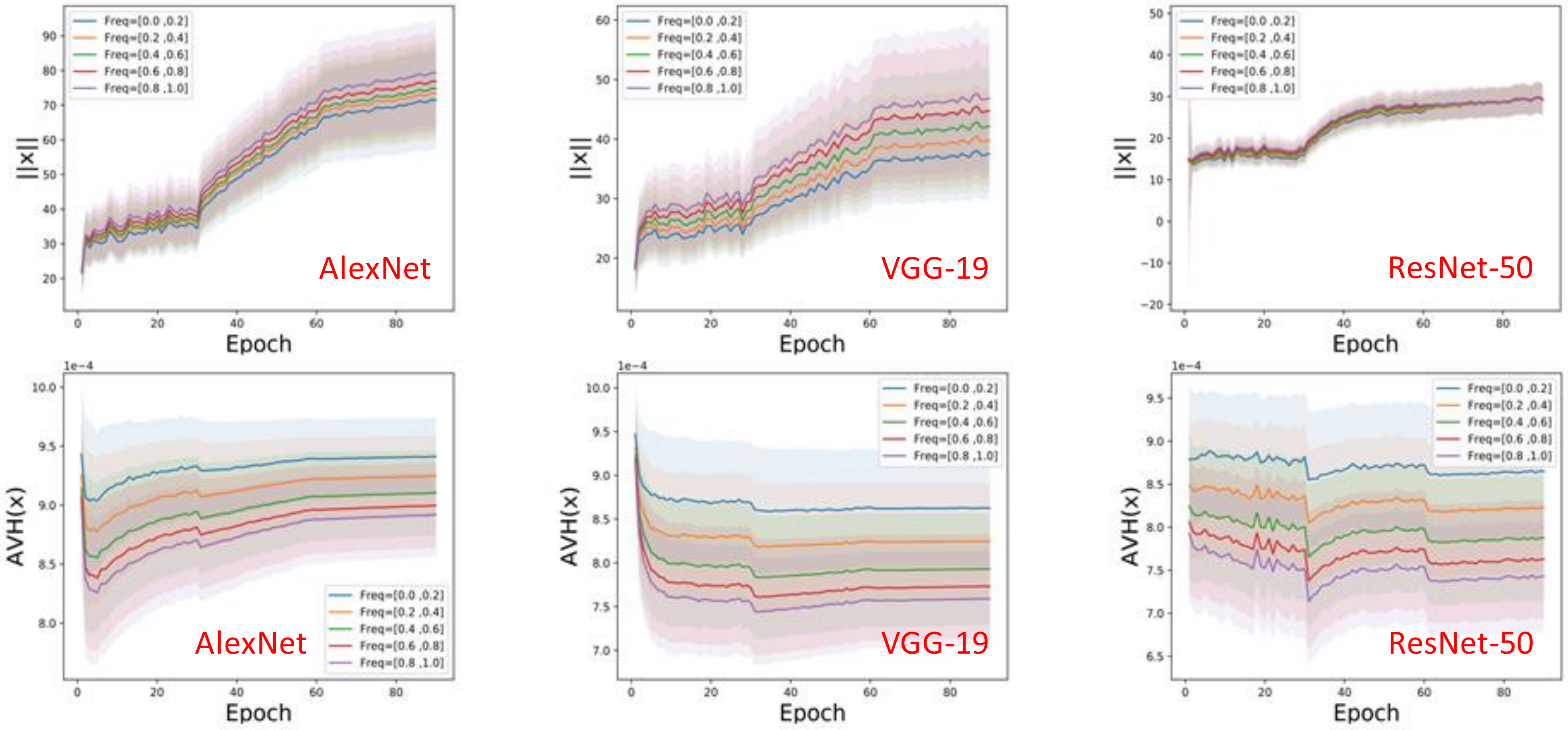
# Main Discoveries

**Discovery 4** - Correlation between AVH and human selection freq holds across models



# Main Discoveries

**Discovery 5** - Correlation between norm and human selection frequency is not consistent



# Conjecture on training dynamic of CNNs

- Softmax cross-entropy loss will first optimize the angles among different classes while the norm will fluctuate and increase very slowly.
- The angles become more stable and change very slowly while the norm increases rapidly.
- Easy examples: the angles get decreased enough for correct classification, the softmax cross-entropy loss can be well minimized by increasing the norm.
- Hard examples: the plateau is cause by unable to decrease the angle to correctly classify examples or increase the norms otherwise hurting loss.

# Application I: Self-Training for Domain Adaptation

VisDA17 Dataset

Car

Source Domain (Labeled)

Adaptation


Target Domain (Unlabeled)

## CBST

$$\hat{y}_t^{(k)*} = \begin{cases} 1, \text{ if } k = \arg\max_c \{\frac{p(c|\mathbf{x}_t;\mathbf{w})}{\lambda_c}\} \\ \quad \text{and} \quad p(k|\mathbf{x}_t;\mathbf{w}) > \lambda_k \\ 0, \text{ otherwise} \end{cases}$$

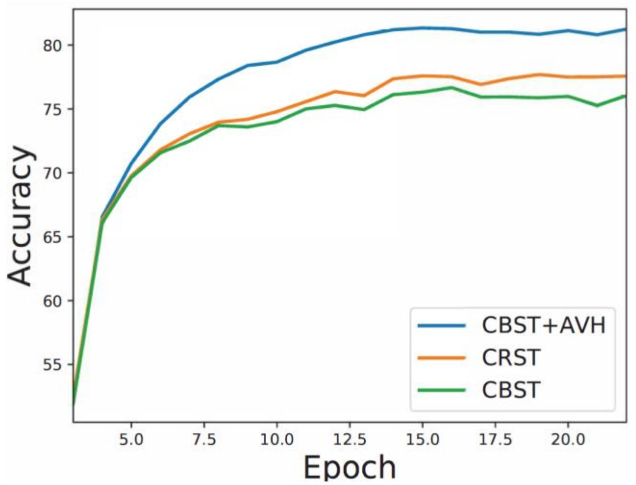
## CBST + AVH

$$\mathcal{AVC}(c|\mathbf{x};\mathbf{w}) = \frac{\pi - \mathcal{A}(\mathbf{x},\mathbf{w}_c)}{\sum_{k=1}^{K}(\pi - \mathcal{A}(\mathbf{x},\mathbf{w}_k))}$$

$$\hat{y}_t^{(k)*} = \begin{cases} 1, \text{ if } k = \arg\max_c \{\frac{p(c|\mathbf{x}_t;\mathbf{w})}{\lambda_c}\} \\ \quad \text{and } \mathcal{AVC}(k|\mathbf{x}_t;\mathbf{w}) > \beta_k \\ 0, \text{ otherwise} \end{cases}$$

Improved selection

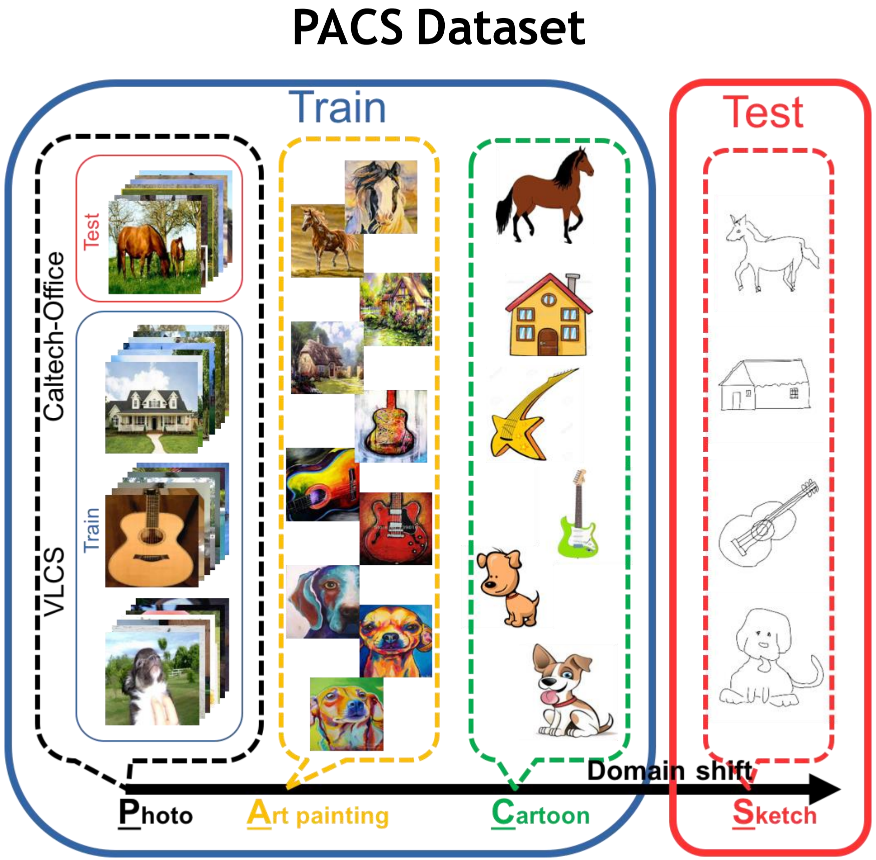
# Application I: Self-Training for Domain Adaptation




Examples chosen by AVH but not Softmax

| Method | Aero | Bike | Bus | Car | Horse | Knife | Motor | Person | Plant | Skateboard | Train | Truck | Mean |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Source (Saito et al., 2018) | 55.1 | 53.3 | 61.9 | 59.1 | 80.6 | 17.9 | 79.7 | 31.2 | 81.0 | 26.5 | 73.5 | 8.5 | 52.4 |
| MMD (Long et al., 2015b) | 87.1 | 63.0 | 76.5 | 42.0 | 90.3 | 42.9 | 85.9 | 53.1 | 49.7 | 36.3 | **85.8** | 20.7 | 61.1 |
| DANN (Ganin et al., 2016) | 81.9 | 77.7 | 82.8 | 44.3 | 81.2 | 29.5 | 65.1 | 28.6 | 51.9 | 54.6 | 82.8 | 7.8 | 57.4 |
| ENT (Grandvalet & Bengio, 2005) | 80.3 | 75.5 | 75.8 | 48.3 | 77.9 | 27.3 | 69.7 | 40.2 | 46.5 | 46.6 | 79.3 | 16.0 | 57.0 |
| MCD (Saito et al., 2017b) | 87.0 | 60.9 | **83.7** | 64.0 | 88.9 | 79.6 | 84.7 | 76.9 | 88.6 | 40.3 | 83.0 | 25.8 | 71.9 |
| ADR (Saito et al., 2018) | 87.8 | 79.5 | **83.7** | 65.3 | **92.3** | 61.8 | **88.9** | 73.2 | 87.8 | 60.0 | 85.5 | 32.3 | 74.8 |
| Source (Zou et al., 2019) | 68.7 | 36.7 | 61.3 | **70.4** | 67.9 | 5.9 | 82.6 | 25.5 | 75.6 | 29.4 | 83.8 | 10.9 | 51.6 |
| CBST (Zou et al., 2019) | 87.2 | 78.8 | 56.5 | 55.4 | 85.1 | 79.2 | 83.8 | 77.7 | 82.8 | **88.8** | 69.0 | **72.0** | 76.4 |
| CRST (Zou et al., 2019) | 88.0 | 79.2 | 61.0 | 60.0 | 87.5 | 81.4 | 86.3 | 78.8 | 85.6 | 86.6 | 73.9 | 68.8 | 78.1 |
| Proposed | **93.3** | **80.2** | 78.9 | 60.9 | 88.4 | **89.7** | **88.9** | **79.6** | **89.5** | 86.8 | 81.5 | 60.0 | **81.5** |

# Application II: AVH Loss for Domain Generalization




$$\mathcal{L}_{AVH} = \sum_i \frac{\exp\left(s(\pi - \mathcal{A}(\mathbf{x}_i, \mathbf{w}_{y_i}))\right)}{\sum_{k=1}^{K} \exp\left(s(\pi - \mathcal{A}(\mathbf{x}_i, \mathbf{w}_k))\right)}$$

| Method | Painting | Cartoon | Photo | Sketch | Avg |
|---|---|---|---|---|---|
| AlexNet (Li et al., 2017) | 62.86 | 66.97 | 89.50 | 57.51 | 69.21 |
| MLDG (Li et al., 2018) | 66.23 | 66.88 | 88.00 | 58.96 | 70.01 |
| MetaReg (Balaji et al., 2018) | **69.82** | 70.35 | **91.07** | 59.26 | **72.62** |
| Feature-critic (Li et al., 2019) | 64.89 | **71.72** | 89.94 | **61.85** | 72.10 |
| Baseline CNN-9 | 66.46 | 67.88 | 89.70 | 51.72 | 68.94 |
| CNN-9 + AVH | **71.56** | **69.25** | **89.93** | **60.86** | **72.90** |

# Thanks You!